Math 213, Spring 2024 Common Exam 1 February 7, 2024

Instructions: (100 Points Total) Please answer questions in the exam booklets. Show all work for full credit. Use a second booklet if required or the back pages of your current booklet. This is a closed book exam. Cheat sheets, calculators, cell phones, tablets, etc. are not permitted.

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Problem 1. (15 points) Let

$$\vec{u} = \langle 3, -1, 0\rangle = 3\vec{i} - \vec{j}\,, \qquad \vec{v} = \langle 4, 4, -2\rangle = 4\vec{i} + 4\vec{j} - 2\vec{k}\,,$$

(a) (3 points) Find $\hat{u}$.
(b) (6 points) Find $\text{proj}_{\vec{v}}\,\vec{u}$.
(c) (6 points) Write $\vec{u}$ as a linear combination of a vector parallel to $\vec{v}$ and a vector perpendicular to $\vec{v}$.

Problem 2. (10 points) Consider the planes:

$$x + 2y + z = 1\,, \qquad x - y + 2z = -7\,.$$

Find a parametric equation for the line of intersection of the two planes.

Problem 3. (15 points) Consider the curve $\vec{r}(t) = \frac{1}{3}t^3\,\vec{i} + \frac{1}{2}t^2\,\vec{j}$ where $t \geq 0$.
(a) (10 points) Find the length of the curve for values of $0 \leq t \leq 1$.
(b) (5 points) Find the unit tangent vector to the curve at the point $(4, 2)$.

Problem 4. (10 points) Let

$$\vec{u} = \vec{i} + \vec{j} - \vec{k}\,, \qquad \vec{v} = 2\vec{i} + \vec{j} + \vec{k}\,, \qquad \vec{w} = -\vec{i} - 2\vec{j} + 3\vec{k}\,.$$

(a) (5 points) Find the area of the parallelogram with sides given by the vectors $\vec{u}$ and $\vec{v}$.
(b) (5 points) Find the volume of the parallelepiped with sides given by the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$.

Problem 5. (10 points)
(a) (5 points) Identify the following quadratic surface and state its center (x_0, y_0, z_0): $x^2 + y^2 + z^2 - 6y + 8z = 0$.
(b) (5 points) Identify the following quadratic surface and state the line (e.g., $\vec{r}(t) = \vec{v}t + \vec{r}_0$) corresponding to its axis of symmetry: $x = y^2 + z^2$.

Problem 6. (15 points) Consider the following line ℓ and plane P:

$$\ell: \quad \vec{r}(t) = (2, 2, 1)t + (3, 0, 0)\,, \qquad P: \quad x + 3y - z = -4\,.$$

(a) (5 points) Find the point P_0 where ℓ intersects P.
(b) (10 points) Find the line that intersects P_0 and is normal to P.

Problem 7. (10 points) Find the distance from the point $(6, 1, 0)$ to the plane $x - 2y + z = 0$.

Problem 8. (15 points) Suppose that a particle is constrained to a wire and follows the path $\vec{r}(t) = \cos(t)\vec{i} + 5\sin t\vec{j}$.
(a) (7 points) Find the velocity and acceleration of the particle at $t = \pi/4$.
(b) (8 points) If the particle were to fly off (in a straight line) the wire at $t = \pi/4$, where would the particle be at time $t = \pi/2$?